

Plato, one of the greatest Greek philosophers and mathematicians, born in 428 BC, suggested that 5040 was the perfect number of citizens in a state, for various reasons including:

- it can be divided by all the numbers to 10 and 12 (and its near neighbour 5038 divides by 11)
- it can be divided by 12 twice over (meaning that the number 144, a "gross" is a factor).

5040 has been quoted as an important number in some types of numerology. The sum of the radii of the Earth and Moon in miles is $3960 + 1080 = 5040$ (in round figures). Twice this figure (the sum of the diameters, or 10,080) is also the number of minutes in a week ($7 \text{ days} \times 24 \text{ hours} \times 60 \text{ minutes} = 10,080$).

Plato was most intrigued that the number 5,040 had 60 multiples which equaled the base system in ancient Greece.

Can you find all of the factors of 5,040 using
Kramer's
factoring methodology?

- remember, the first factor set is 1 and the number itself.
- if the number is even, as is 5,040, we double the number 1 and take half of the number 5,040 until we hit an odd number.
- then we divide into 5,040 the next odd factor of the remaining odd number.
- when we have used all of the odd factors, we look at the products of the prime factors to see if we need to use any of those numbers (for example, the prime factors of 5,040 are 2, 3, 5, and 7; we find that the products of 3 and 5 = 15 and 3 and 7 = 21 have not been used.

1

5,040

**Let's see if you can find all
60 factors, or 30 factor sets.**

**It is amazing that Dover and
Sherborn each have close to
Plato's number in its number
of citizens. Although
Wellesley has close to 30,000
citizens, its 6 school districts
are broken up into very close
to Plato's optimal number.**

1	5,040	7 720	
2 2,520		14	360
4	1,260	28	180
8	630	56	90
16	315	112	45
3 1,680		9 560	
6	840	18	280
12	420	36	140
24	210	72	70
48	105	144	35
5 1,008		15 336	
10	504	30	168
20	252	60	84
40	126	120	42
80	63	240	21

The bold face numbers 2, 3, 5, 7, 9 and 15 are the odd or prime factors of 5,040. If they are not prime factors, they are products of the prime factors.